(Pages : 6) **N – 1295**

**Reg. No. : ..............................**

**Name : ..................................**

## Sixth Semester B.Sc. Degree Examination, April 2022

## First Degree Programme under CBCSS

## Mathematics

## Core Course XIII

## MM 1645 : INTEGRAL TRANSFORMS

## (2018 & 2019 Admission)

Time : 3 Hours Max. Marks : 80

PART – A

All the **first ten** questions are compulsory. They carry **1** mark each.

1. Find the Laplace transform of $f(t) = \cos 2t$.

2. If $L[f(t)] = F(s)$, then $L[f'(t)] =$ ————

3. Find the inverse Laplace transform of $\frac{1}{s^2 + 9}$.

4. Define unit step function.

5. Write $L\{f''(t)\}$ in terms of $L(f)$, $f(0)$ and $f'(0)$.

**P.T.O.**

6. Define Fourier sine transform of a function $f(x)$.

7. What is the standard form of Fourier series for an odd function?

8. If $f(x)$ and $g(x)$ have period $p$ then find the period of $a\, f(x)+b\, g(x)$ with any constant $a$ and $b$.

9. If $f(x)$ has period $p$ then find the period of $f(nx)$.

10. Find the fundamental period of the function $\cos\left(\frac{x}{5}\right)$.

**(10 × 1 = 10 Marks)**

PART – B

Answer **any eight** questions. Each question carries **2** marks.

11. Find the Laplace transform of $f(t) = t \cos 4t$.

12. Find the Laplace transform of $f(t) = \cos 3t \cos 2t$.

13. Find $L\left(e^{-3t} \cos 2t\right)$.

14. Evaluate $L^{-1}\left[\frac{2}{(s+4)^3}\right]$.

15. Find $L^{-1}\left(\frac{1}{(s+1)(s+2)}\right)$.

16. Is $L[f(t)\, g(t)] = L[f(t)]\, L[g(t)]$? Explain.

17. Solve $y'' + y' - 6y = 0$, $y(0) = 1$, $y'(0) = 1$.

18. Find the convolution of $t$ and $e^{-t}$.

19. Write down the Euler formulae for calculating the Fourier coefficients of function $f(x)$ of period $2\pi$.

20. Find the Fourier series of $f(x) = x$ for $0 < x < 2\pi$.

21. Find the Fourier transform of $f(x)$ given by $f(x) = 3$ if $-2 \le x \le 2$ and $f(x) = 0$, otherwise.

22. Find the Fourier sine series for the function $f(x)$, where $f(x) = \pi - x$ in $0 < x < \pi$.

23. Derive the relation between $F[f'(x)]$ and $F[f(x)]$.

24. State the convolution theorem of Fourier transform.

25. Show that sum of two odd functions is odd.

26. Check whether the following functions are odd or even

(a) $e^{-|x|}$ (b) $x^3 \cos nx$.

**(8 × 2 = 16 Marks)**

PART – C

Answer **any six** questions. Each question carries **4** marks.

27. Find the Laplace transform of the function $f(t) = \begin{cases} t, t \ge 2 \\ 0, t < 2 \end{cases}$.

28. Find the inverse transform of $(3s - 137) / (s^2 + 2s + 401)$.

29. Solve $y'' + 3y' + 2y = r(t) = u(t-1) - u(t-2)$, $y(0) = 0$, $y'(0) = 0$.

30. Find the Laplace transform of the integral $\int_0^t t e^{-4t} \sin 3t \, dt$.

31. Find the inverse Laplace transform of $\frac{s\left(e^{-3s}-e^{-7s}\right)}{s^2+25}$.

32. Find Fourier cosine transform of $f(x)=\begin{cases}1 & 0<x<a\\ 0 & x>a\end{cases}$

33. Discuss the Fourier series representation of $f(x)=\sin\left(\frac{1}{x}\right)$ in $0<x<1$ treating $f(x)$ as a periodic function with period 1.

34. Expand the function defined by $f(x)=\begin{cases}0 & \text{for } -2<x<0\\ x & \text{for } 0\leq x<2\end{cases}$ as a Fourier series on $[-2, 2]$.

35. For the function $f(x)=0$ when $x<-\pi, x>\pi$ and $f(x)=-1$ if $-\pi<x<0$ and 1 for $0<x<\pi$.

(a) Find Fourier integral representation of $f(x)$.

(b) What will be the value of the integral at $x=-\pi$.

36. Show that the Fourier transform is a linear operator.

37. Express $f(x)=\begin{cases}\frac{1}{2} & \text{if } 0<x<\pi\\ 0 & \text{if } x>\pi\end{cases}$ as a Fourier sine integral.

38. Find Fourier cosine transform of $f(x)=\begin{cases}x, & 0<x<a\\ 0, & x>a\end{cases}$.

**(6 × 4 = 24 Marks)**

## PART – D

Answer **any two** questions. Each question carries **15** marks.

39. (a) If $L[f(t)] = F(s)$ Show that $L[f(t-a)u(t-a)] = e^{-as} F(s)$.

(b) Using Laplace transform solve $y'' + 4y' + 3y = e^{-t}$, $y(0) = y'(0) = 1$.

40. (a) State the convolution theorem for Laplace transforms.

(b) Use it to find the inverse Laplace transform of $\dfrac{s}{(s-1)(s^2+4)}$. Verify the result by finding the inverse using partial fraction technique.

41. If $f(x) = e^{-kx}$ $(x>0, k>0)$. Then

(a) Find the Fourier sine and cosine transform of $f(x)$

(b) Prove that $\int_0^{\infty} \dfrac{x \sin mx}{x^2+k^2} dx = \dfrac{\pi}{2} e^{-km}$.

42. Find a Fourier series to represent $f(x) = x - x^2$ in $(-\pi, \pi)$ and hence show that $\dfrac{1}{1^2} - \dfrac{1}{2^2} + \dfrac{1}{3^2} - \dfrac{1}{4^2} + \ldots = \dfrac{\pi^2}{12}$.

43. (a) Represent $f(x) = e^{-kx}$ $(x>0, k>0)$ as a Fourier cosine integral

(b) Find the Fourier transform of $f(x)$, where

$$f(x) = \begin{cases} 1 & \text{if } |x| < 1 \\ 0 & \text{otherwise} \end{cases}$$

44. (a) Obtain the half range Fourier cosine series for the function

$$f(x)=\begin{cases}\cos x, 0 < x < \pi/2 \\ 0, \ \pi/2 < x < \pi\end{cases} \text{ in } (0, \pi).$$

(b) Find a Fourier series that represents $f(x)=|x|$ in $[-\pi, \pi]$ and deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\ldots=\frac{\pi^2}{8}$.

**(2 × 15 = 30 Marks)**

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**Reg. No. : ..............................**

**Name : ..................................**

**Sixth Semester B.Sc. Degree Examination, April 2022**

**First Degree Programme under CBCSS**

**Mathematics**

**Core Course XII**

**MM 1644 : LINEAR ALGEBRA**

**(2018 and 2019 Admission)**

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. Each carries **1** mark.

1. Write the following system of equations in column form.

$$2x+3y=10$$
$$4x-5y=11$$

2. Describe the intersection of the three planes $u+v+w+z=6$, $u+w+z=4$ and $u+w=2$, all in four dimensional space.

3. Define a skew symmetric matrix.

4. Define column space of a matrix.

5. If $v_1, v_2, \ldots v_n$ are linearly independent, the space they span has dimension __________

6. Write the rotation matrix that turns all vectors in the $xy$ plane through 90°.

P.T.O.

7. If a 4 by 4 matrix A has $\det A = \frac{1}{2}$, find $\det(2A)$.

8. State whether true or false : If $\det A = 0$, then at least one of the cofactors must be zero.

9. The eigen values of a projection matrix are ————.

10. State principal axis theorem.

## SECTION – B

Answer **any eight** questions. Each carries **2** marks.

11. Draw the two pictures in two planes for the equations $x - 2y = 0, x + y = 6$.

12. Find the inner product $\begin{bmatrix} 1 & -2 & 7 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 7 \end{bmatrix}$.

13. Write a 2 by 2 matrix $A$ such that $A^2 = -I$.

14. Prove that $(AB)^{-1} = B^{-1}A^{-1}$.

15. Describe the column space of the matrix $A = \begin{bmatrix} 0 & 0 & 3 \\ 1 & 2 & 3 \end{bmatrix}$.

16. Determine whether the vectors (1, 0, 0), (–1, 2, 1) and (2, 1, 1) are linearly independent.

17. Describe the subspace of $R^3$ spanned by the three vectors (0, 1, 1), (1, 1, 0) and (0, 0, 0).

18. Prove that a linear transformation leaves the zero vector fixed.

19. Find the determinant of $A = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 2 \end{bmatrix}$.

20. Using the big formula, compute the determinant of $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 3 & 2 & 1 \end{bmatrix}$ from six terms. Are rows independent.

21. Find the area of the parallelogram with edges $v = (3, 2)$ and $w = (1, 4)$.

22. Suppose the permutation $p$ takes (1, 2, 3, 4, 5) to (5, 4, 1, 2, 3). What does $p^2$ do to (1, 2, 3, 4, 5)?

23. Suppose A and B have the same eigen values $\lambda_1, \ldots \lambda_m$ with the same independent eigen vectors $x_1, \ldots x_m$. Show that $A = B$.

24. Describe all matrices $S$ that diagonalize the matrix $A = \begin{bmatrix} 4 & 0 \\ 1 & 2 \end{bmatrix}$.

25. Prove that every third Fibonacci number in 0, 1, 1, 2, 3,... is even.

26. Prove that if $A = A^H$, then every eigen value is real.

## SECTION – C

Answer **any six** questions. Each carries **4** marks.

27. Find a coefficient 'b' that makes the following system singular. Then choose a right hand side 'g' that makes it solvable. Find two solutions in that singular case.

28. The parabola $y = a + bx + cx^2$ goes through the points $(x, y) = (1, 4), (2, 8)$ and $(3, 14)$. Find and solve a matrix equation for the unknowns $(a, b, c)$.

29. Solve $Lc = b$ to find $c$. Then solve $Ux = c$ to find $x$.

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix},\ U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } b = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}.$$

30. Reduce to echelon form and find the rank $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 5 & 5 & 8 \end{bmatrix}$.

31. What are the special solutions to $Rx = 0$ and $R^T y = 0$ for the given $R$. $R = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

32. Find the range and kernel of $T$.

$$T(v_1, v_2, v_3) = (v_1 + v_2, v_2 + v_3, v_1 + v_3)$$

33. By applying row operations, produce an upper triangular matrix $u$ and compute $\det\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 6 & 6 & 1 \\ -1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 7 \end{bmatrix}$.

34. Find $x$, $y$ and $z$ by Cramer's rule.

$$x - 3y + z = 2$$
$$3x + y + z = 6$$
$$5x + y + 3z = 3$$

35. If every row of a matrix $A$ adds to 1, prove that $\det(A - I) = 0$. Show by an example that this doesn't imply $\det A = 1$.

36. Find the eigen values and eigen vectors of $A=\begin{bmatrix}3 & 4 & 2\\ 0 & 1 & 2\\ 0 & 0 & 0\end{bmatrix}$.

37. Prove that Diagonalizable matrices share the same eigen vector matrix $S$ if and only if $AB=BA$.

38. Write out the matrix $A^H$ and compute $C=A^H A$ if $A=\begin{bmatrix}1 & i & 0\\ i & 0 & 1\end{bmatrix}$.

## SECTION – D

Answer **any two** questions. Each carries **15** marks.

39. (a) Reduce the system to upper triangular form and solve by back substitution for $z$, $y$, $x$.

$$2x-3y=3$$
$$4x-5y+z=7$$
$$2x-y+3z=5$$

(b) Use Gauss Jordan method to find $A^{-1}$

$$A=\begin{bmatrix}2 & 1 & 1\\ 4 & -6 & 0\\ -2 & 7 & 2\end{bmatrix}$$

40. (a) What are $L$ and $D$ for the matrix $A$. What is $U$ in $A=Lu$ and what is the new $U$ in $A=LDu$?

$$A=\begin{bmatrix}2 & 4 & 8\\ 0 & 3 & 9\\ 0 & 0 & 7\end{bmatrix}$$

(b) Compute $LDL^T$ factorisation of $\begin{bmatrix}1 & 2\\ 2 & 8\end{bmatrix}$.

41. (a) Find the value of $c$ that makes it possible to solve $Ax = b$ and solve it :

$$u + v + 2w = 2$$
$$2u + 3v - w = 5$$
$$3u + 4v + w = c$$

(b) Find the dimensions of the column space and row space of $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 3 & 1 \\ 3 & 1 & -1 \end{bmatrix}$.

42. (a) Find the dimensions and a basis for the four fundamental subspaces for $A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$.

(b) Check whether the mapping $T : R^2 \to R^2$ given by

$T(v_1, v_2) = (v_1 + v_2, v_1 - v_2, v_2)$ is linear.

43. Find the determinant and compute the cofactor matrix $C$. Verify that $AC^T = (\det A)I$. What is $A^{-1}$?

44. (a) Test the Cayley Hamilton theorem on $A = \begin{bmatrix} 0 & 3 & -2 \\ -3 & 0 & 1 \\ 2 & -1 & 0 \end{bmatrix}$.

(b) Find a suitable $P$ such that the matrix $B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ is similar to $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

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# N – 1285

**Reg. No. : ..............................**

**Name : ..................................**

## Sixth Semester B.Sc. Degree Examination, April 2022

## First Degree Programme Under CBCSS

## Mathematics

## Core Course IX

## MM 1641 – REAL ANALYSIS – II

## (2018 & 2019 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – A

All the first ten questions are compulsory. They carry 1 mark each.

1. Evaluate $\lim\limits_{x\to\infty}\left(1+\dfrac{1}{n}\right)^n$.

2. State true or false: A function that is continuous on a compact set $K$ is uniformly continuous on $K$.

3. Define a bounded function.

4. Determine the points of discontinuity of the Dirichlet's function.

5. State Rolle's theorem.

6. State intermediate value property.

7. When do you say that a function is differentiable on an interval?

P.T.O.

8. State true or false: If $f$ is differentiable in $[a, b]$ and $f'(x) = 0$ for all $x \in (a, b)$, then $f$ is continuous.

9. Define lower integral of a function $f$.

10. Compute $\int_0^3 [x]\, dx$, where $[x]$ denotes the greatest integer function.

## SECTION – B

Answer **any eight** questions. Each question carries **2** marks.

11. Show that the limit $\lim_{x \to 2} \frac{|x-2|}{x-2}$ doesn't exist.

12. Let $f$ and $g$ be real valued functions then prove that

$$\lim_{x \to c} \{f(x) + g(x)\} = \lim_{x \to c} f(x) + \lim_{x \to c} g(x).$$

13. Show that $|x|$ is continuous everywhere.

14. Let $[x]$ denote the largest integer containing in $x$ and $(x) = x - [x]$ denote the fractional part of $x$. What discontinuity do the function $(x)$ has?

15. If, $f$, $g$ be two functions continuous at a point $c$ then the function $f$-$g$ is also continuous at $c$.

16. Show that the function $f(x) = x^2$ is uniformly continuous on $[-1, 1]$.

17. Suppose that $\{x_n\}$ is a Cauchy sequence in $R$. Prove that $f(x_n)$ is a Cauchy sequence where $f$ is a uniformly continuous function.

18. State Squeez theorem.

19. Give an example to show that continuous function need not be differentiable.

20. If $f$ is differentiable in $(a, b)$ and $f'(x) \geq 0$ for all $x \in (a, b)$, show that $f$ is monotonically increasing.

21. Suppose $f$ and $g$ are defined on $[a, b]$ and are differential at a point $x \in [a, b]$. Prove that $f + g$ is differentiable.

22. Show that $\underline{\int_a^b} f dx \le \overline{\int_a^b} f dx$.

23. Show that the function $f(x)$ defined on $\mathbf{R}$ by $f(x) = \begin{cases} x \text{ if } x \text{ is irrational} \\ -x \text{ if } x \text{ is rational} \end{cases}$ is continuous only at $x = 0$.

24. If $P_1$ and $P_2$ are any two partitions of $[a,b]$ ,then $L(f,P_1) \le U(f,P_2)$.

25. If a function $f$ is integrable on $[a,b]$ and $m \le f(x) \le M$ for $x \in [a,b]$, prove that $m(b-a) \le \int_a^b f \le M(b-a)$.

26. Show that if $f$ and $g$ are bounded and integrable on $[a,b]$ such that $f \le g$ then $\int_a^b f\ dx \le \int_a^b g\ dx$.

## SECTION – C

Answer **any six** questions. Each question carries **4** marks.

27. Show that $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$ does not exist.

28. Assume $f$ and g are defined on all of $R$ and that $\lim_{x \to p} f(x) = q$ and $\lim_{x \to q} g(x) = r$. Give an example to show that it may not be true that $\lim_{x \to p} g\ (f(x)) = r$.

29. Prove that a function which is continuous on a closed interval is uniformly continuous on that interval.

30. If $f$ is a real differentiable function on $[a,b]$ and suppose $f'(a) < \lambda < f'(b)$ then prove that there is a point $x \in (a,b)$ such that $f'(x) = \lambda$.

31. State and prove extreme value theorem.

32. Prove that if $f$ is differentiable on an open interval in $(a, b)$ and $f$ attains a maximum value at some point $c$ in $(a, b)$, then $f'(c) = 0$.

33. Show that $\int_3^4 f dx = \frac{41}{2}$ where $f(x) = 5x + 3$.

34. If $g : A \to R$ is differentiable on an interval $A$ and satisfies $g'(x) = 0$ for all $x \in A$, then prove that $g(x) = k$ for some constant $k \in R$.

35. Prove that a continuous function in a closed interval is integrable in that interval.

36. Prove that if $f$ is monotonic in $[a, b]$ then $f$ is integrable in $[a, b]$.

37. If $f$ is bounded and integrable in $[a, b]$, prove that there exists a number $\mu$ lying between $a$ and $b$ such that $\int_a^b f(x)\, dx = \mu(b - a)$.

38. Assume $f$ is integrable function on the interval $[a, b]$, then show that $|f|$ is also integrable and $\left|\int_a^b f\right| \leq \int_a^b |f|$.

## SECTION – D

Answer **any two** questions. Each question carries **15** marks.

39. State and prove intermediate value theorem. Is the converse true? Justify.

40. Define Lipschitz functions. Show that every Lipschitz function is uniformly continuous. Is the converse statement true? Justify.

41. State and prove chain rule for differentiation.

42. (a) State and prove Mean value theorem.

    (b) Prove that if $f$ is continuous in $[a, b]$, then $f$ is integrable in $[a, b]$.

43. If $f : [a, b] \to R$ is bounded, and $f$ is integrable on $[c, b]$ for all $c \in (a, b)$, then prove that $f$ is integrable on $[a, b]$.

44. State and prove fundamental theorem of calculus.

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**Reg. No. : ..............................**

**Name : ..................................**

**Sixth Semester B.Sc. Degree Examination, April 2022**

**First Degree Programme under CBCSS**

**Mathematics**

**Core Course – XI**

**MM 1643 : ABSTRACT ALGEBRA – RING THEORY**

**(2018 & 2019 Admission)**

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. Each carries **1** mark.

1. Which are the units of $Z$?
2. Define a zero divisor.
3. Give an example of a non commutative ring.
4. Define an ideal of a ring.
5. State the first isomorphism theorem for rings.
6. Is the ring $2z$ is isomorphic to the ring $3z$?
7. Define the content of a nonzero polynomial.
8. Define an irreducible element in an integral domain.
9. Find the norm of $a+b\sqrt{d}$, where $a$ and $b$ are integers.
10. Define a Noetherian domain.

**(10 × 1 = 10 Marks)**

P.T.O.

## SECTION – B

Answer **any eight** questions. Each carries **2** marks.

11. If $a$ is an element in a ring $R$, then prove that $a0 = 0a = 0$

12. Prove that if a ring element has a multiplicative inverse, then it is unique.

13. Let $a$, $b$ and $c$ belong to an integral domain. Prove that if $a \neq 0$ and $ab = ac$, then $b = c$.

14. Show that 0 is the only nilpotent element in an integral domain.

15. Prove that the ideal $\langle x^2 + 1 \rangle$ is maximal in $R[x]$.

16. Prove that the only ideals of a field $F$ are {0} and $F$ itself.

17. Determine all ring homomorphisms from $z$ to z.

18. Find a polynomial with integer coefficients that has $\frac{1}{2}$ and $\frac{-1}{3}$ are zeros.

19. Give an example of a field that properly contains the field of complex numbers $c$.

20. Let $F$ be a field and let $a$ be a non zero element of $F$. Show that if $f(x+a)$ is irreducible over $F$, then $f(x)$ is irreducible over $F$.

21. Construct a field of order 25.

22. Find the kernel of the ring homomorphism $\varphi$ from $R[x]$ onto $c$ given by $f(x) \rightarrow f(i)$.

23. Prove that in an integral domain, every prime is irreducible.

24. Show that $z[x]$ is not a principal ideal domain.

25. Find $q$ and $r$ in $z[i]$ such that $3 - 4i = (2+5i)q + r$ and $d(r) < d(2+5i)$.

26. Show that for any non trivial ideal $I$ of $z[i]$, $\frac{z[i]}{I}$ is finite.

**(8 × 2 = 16 Marks)**

## SECTION – C

Answer **any six** questions. Each carries **4** marks.

27. Let $R$ be a ring. Prove that $a^2 - b^2 = (a+b)\,(a-b) \forall\ a, b \in R$ if and only if $R$ is commutative.

28. Let $R$ be a ring with unity 1. Show that if 1 has order $n$ under addition, then the characteristic of $R$ is $n$.

29. Show that a finite commutative ring with no zero divisors and at least two elements has a unity.

30. Find all solutions of the equation $x^3 - 2x^2 - 3 = 0$ in $z_{12}$.

31. Let $\varphi$ be a ring homomorphism from a ring $R$ to a ring $S$. Let $A$ be a subring of $R$. Prove that if $A$ is an ideal and $\varphi$ is onto $S$, then $\varphi(A)$ is an ideal.

32. Let $F$ be a field. If $f(x) \in F[x]$ and $\deg f(x)$ is 2 or 3, then prove that $f(x)$ is reducible over $F$ if and only if $f(x)$ has a zero in $F$.

33. Find the quotient and remainder upon dividing $f(x) = 3x^4 + x^3 + 2x^2 + 1$ by $g(x) = x^2 + 4x + 2$, where $f(x)$ and $g(x)$ belong to $z_5(x)$.

34. Check whether $f(x) = 21x^3 - 3x^2 + 2x + 9$ is an irreducible polynomial over $Q$.

35. Show that 7 is irreducible in the ring $z[\sqrt{5}]$.

36. Prove that every Euclidean Domain is an Principal Ideal Domain.

37. Prove that in a PID, any strictly increasing chain of ideals $I_2 \subset I_3 \subset \ldots$ must be finite in length.

38. Let $D$ be a Euclidean Domain with measure $d$. Show that if $a$ and $b$ are associates in $D$, then $d(a) = d(b)$.

**(6 × 4 = 24 Marks)**

## SECTION – D

Answer **any two** questions. Each carries **15** marks.

39. (a) Let $d$ be an integer. Prove that $z(\sqrt{d})-\{a+b\sqrt{d}\mid a, b\in z\}$ is an integral domain.

    (b) Define center of a ring $R$. Prove that the center of a ring is a subring.

40. Let $R$ be a ring and let $A$ be a subring of $R$. Prove that the set of cosets $\{r+A\mid r\in R\}$ is a ring under the operations $(s+A)+(t+A)=s+t+A$ and $(s+A)\,(t+A)=st+A$ if and only if $A$ is an ideal of $R$.

41. State and prove division algorithm for $F[x]$.

42. Let $F$ be a field. Prove that $F[x]$ is a principal ideal domain.

43. Let $F$ be a field and let $p(x)\in F[x]$. Then prove that $\langle p(x)\rangle$ is a maximal ideal in $F[x]$ if and only if $p(x)$ is irreducible over $F$.

44. Show that the ring $Z[\sqrt{-5}]$ is not a Unique Factorisation Domain.

**(2 × 15 = 30 Marks)**

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**Reg. No. : ...............................**

**Name : ...................................**

**Sixth Semester B.Sc. Degree Examination, April 2022**

**First Degree Programme Under CBCSS**

**Mathematics**

**Core Course X**

**MM 1642 – COMPLEX ANALYSIS II**

**(2018 & 2019 Admission)**

Time : 3 Hours Max. Marks : 80

PART – A

Answer **all** questions. **Each** question carries **1** mark.

1. When does a complex series diverge?

2. Define Uniform Convergence of a series of functions.

3. Explain the term: Cauchy sequence.

4. Define a zero of order m for a function $f$.

5. What is the formula for

   $\operatorname{Res}(f;z_0)$ if $f(z)=\frac{p(z)}{Q(z)}$, where $P(z)$ and $Q(z)$ are analytic at $z_0$

   and Q has a simple pole at $z_0$, while $P(z_0)\neq 0$.

6. Define Improper integral over $(-\infty,0)$ of a continuous function $f(x)$.

P.T.O.

7. Define p.v. $\int_{-\infty}^{\infty} f(x)dx$.

8. What do you mean by global property of a mapping?

9. Are analytic functions that are non constant in domains being open mappings?

10. Define the mapping : Rotation.

**(10 × 1 = 10 Marks)**

PART – B

Answer **any eight** questions. **Each** question carries **2** marks.

11. Prove that $1+c+c^2+c^3+\cdots=\frac{1}{1-c}$ for $|c|<1$.

12. Find the Maclaurin series for cos $z$.

13. How can we obtain the Taylor series of *fg*, if *f* and *g* are two analytic functions?

14. State a necessary and sufficient condition for the convergence of a sequence of complex numbers.

15. Explain the term: Laurent Series

16. State the necessary and sufficient condition for an analytic function to have a pole of order $m$ at $z_0$.

17. State the Picard's Theorem.

18. Define of a residue of a function with an example.

19. Find the residue at $z=0$ of $f(z)=ze^{\frac{3}{z}}$ using Laurent series.

20. Find the residue at $z=-3i$ of $f(z)=\frac{z+1}{z^2+9}$.

21. Evaluate $p.v. \int_{-\infty}^{\infty} x^3 dx$.

22. Give the statement of Jordan's Lemma.

23. Is $f(z)=z^2$ being locally one to one on a neighbourhood of 0? Justify your answer.

24. Write the statement of Riemann Mapping Theorem.

25. Define magnification and show that it rescales distance.

26. State the symmetry principle of Mobius transformations.

**(8 × 2 = 16 Marks)**

PART – C

Answer **any six** questions. **Each** question carries **4** marks.

27. State the Ratio test and show that $\sum_{j=1}^{\infty}\frac{4^j}{j!}$ converges.

28. If $f(z)$ is analytic at $z_0$, how can we find the Taylor series of $f'(z)$. Using this result, find the Taylor series of cos $z$ from that of $z=\sum_{j=1}^{\infty}\frac{z^{2j-1}}{(2j-1)!}$.

29. Prove that the uniform limit of a sequence of analytic functions defined on a simply connected domain is also analytic.

30. Classify the zeros and singularities of $\frac{\tan z}{z}$.

31. What do you mean by extended complex plane? Classify the behaviour at $\infty$ of $f(z)=\frac{iz+1}{z-1}$.

32. Find the residues at each singularity of $f(z) = \text{cosec}\, z$.

33. State and prove Cauchy Residue Theorem.

34. Prove that if $|f(t)| \le M(t)$ on $a \le t \le b$, then $\left|\int_a^b f(t)dt\right| \le \int_a^b M(t)dt$ where $f(t)$ and $M(t)$ are continuous function defined on $[a, b]$, with $f$ complex and $M$ real valued.

35. Find the integral of $\frac{\sin x}{x}$ over $(0,\infty)$.

36. Prove that if $f(z)$ is analytic at $z_0$, then there is an open disk $D$ centred at $z_0$ such that $f$ is one to one on $D$.

37. Define linear transformation and find the linear transformation that maps the circle $|z-1|=1$ onto the circle $\left|w-\frac{3i}{2}\right|=2$.

38. Show that the composition of two Mobius transforms is another Mobius Transform.

**(6 × 4 = 24 Marks)**

PART – D

Answer **any two** questions. **Each** question carries **15** marks.

39. Define Taylor Series of $f(z)$ around $z_0$. Prove that if $f(z)$ is analytic in the $|z-z_0|<R$ there exists a Taylor Series which converges to $f(z)$ for all $z$ in this disk.

40. Find The Laurent series expansion of $f(z) = \frac{1}{(z-1)(z-2)}$ in

(a) The region $|Z|<1$

(b) The region $1<|Z|<2$

(c) The region $|Z|>2$.

41. Define different singularities of a complex function with examples. Verify the examples with definition.

42. (a) Prove that, if $f(z)$ has a pole of order $m$ at $z_0$, then

$$\operatorname{Res}(f:z_0)=\lim_{z\to z_0}\frac{1}{(m-1)!}\frac{d^{m-1}}{dz^{m-1}}\left[(z-z_0)^m f(z)\right].$$

(b) Using the result find the residue at $z=0$ of $f(z)=\frac{\cos z}{z^2(z-\pi)^3}$.

43. Evaluate $\int_0^{x}\frac{1}{2-\cos\theta}d\theta$.

44. (a) Prove that if $f(z)$ is analytic at $z_0$ and $f'(z_0)\neq 0$ then $f(z)$ is conformal at $z_0$.

(b) Find all Mobius transformations that map unit disks into unit disks.

**(2 × 15 = 30 Marks)**

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**Reg. No. : ..............................**

**Name : ..................................**

**Sixth Semester B.Sc. Degree Examination, April 2022.**

**First Degree Programme under CBCSS**

**Mathematics**

**MM 1661.1 – GRAPH THEORY**

**(2018 & 2019 Admission)**

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer all the questions. Each question carries 1 mark.

1. Define a graph.
2. Draw a complete graph on 6 vertices.
3. Define a bipartite graph.
4. Define adjacency matrix of a graph.
5. State Cayley Theorem.
6. Define cut vertex of a graph.
7. Define an Eulerian graph.
8. Give an example for a polyhedral graph.
9. Draw a non- Hamiltonian graph containing a Hamiltonian path.
10. State Kuratowski's theorem.

**(10 × 1 = 10 Marks)**

P.T.O.

## SECTION – II

Answer **any eight** questions. Each question carries **2** marks

11. Give 2 drawings of $K_{3,3}$ which are isomorphic.

12. Define a regular graph. Give one example.

13. Define an edge deleted subgraph. Give an example.

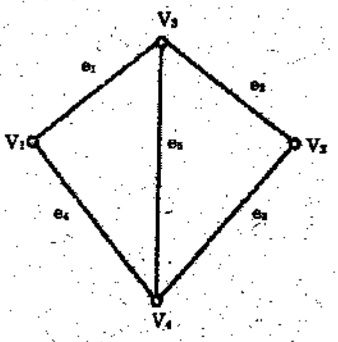
14. Define a tree. Draw all non-isomorphic trees with 5 vertices.

15. Write $\omega(G)$ for a connected and disconnected graph.

16. Let G be a connected graph. If every edge of G is a bridge, then prove that G is a tree.

17. Define a bridge. Give an example.

18. Write the incidence matrix of the following graph




19. Explain Konigsberg Bridge Problem.

20. Explain Chinese Postman Problem in graph theoretical terms.

21. Define closure of a graph. Find the closure of $C_4$.

22. Let $G_1$ and $G_2$ be 2 plane graphs which are both redrawings of the same planar graph. Then prove that $G_1$ and $G_2$ have the same number of faces.

23. Is $K_4$ Hamiltonian. Justify your answer

24. Draw a connected plane graph and verify Euler's Formula for the graph.

25. Draw a graph G satisfy the relation $k(G) = k(G * e)$.

26. Which are the only polyhedral graphs which are regular,

**(8 × 2 = 16 Marks)**

## SECTION – III

Answer **any six** questions. Each question carries **4** marks

27. State and prove first theorem on Graph theory.

28. Define (a) eccentricity of a vertex (b) radius of a graph (c) diameter of a graph (d) Find the radius and diameter of the Peterson Graph

29. Prove that every u-v walk contains a u-v path for any 2 vertices u and v of a graph G.

30. Let G be a graph with n vertices $v_1, v_2, \dots v_n$ and let A denotes the adjacency matrix of G with respect to the listing of vertices. Let k be any positive integer and let $A^k$ denote the matrix multiplication of k copies of A. Prove that the $(i,j)^{th}$ entry *of* $A^k$ is the number of different $v_i - v_j$ walks in G of length k.

31. Let $G$ be a graph with $n \geq 2$ vertices. Then prove that G has at least 2 vertices which are not cut vertices.

32. If a graph $G$ is connected, then prove that it has a spanning tree.

33. Prove that a simple graph is Hamiltonian if and only if its closure is Hamiltonian.

34. Prove that $K_5$ is non planar.

35. Let G be a connected simple planar graph with $n \geq 3$ vertices and e edges, Prove that $e \leq 3n - 6$.

36. Let P be a convex polyhedron and G be its corresponding polyhedral graph. Prove that P and so the graph G has at least one face bounded by a cycle of length n for either n = 3,4 or *5*.

37. Explain Travelling salesman Problem

38. Prove that a connected graph has an Euler trail if and only if it has at most two odd vertices.

**(6 × 4 = 24 Marks)**

SECTION – IV

Answer **any two** questions. Each question carries **15** marks

39. (a) Prove that a tree with $n$ vertices has precisely $n-1$ edges.

    (b) Prove that an edge e of a graph $G$ is a bridge if and only if e is not any part of any cycle in $G$.

40. State and Prove Whitney's Theorem.

41. Let $G$ be a non-empty connected graph with at least 2 vertices. Prove that G is bipartite if and only if G contains no odd cycle.

42. Prove that a connected graph is Euler if and only if degree of every vertex is even.

43. State and Prove Dirac Theorem.

44. (a) Define (i) Subdivision of a graph (ii) Contraction on an edge, with examples.

    (b) Let $G$ be a simple 3-connected graph with at least 5 vertices. Then prove that G has a contractible edge

**(2 × 15 = 30 Marks)**